

## MATHEMATICAL MODELLING OF THE INJECTION PROCESS RUN IN COMMON RAIL SYSTEM BY "MOVING VOLUMETRIC ELEMENT" METHOD

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### **Abstract**

*In the article the mathematical simulation of the injection process run with Common Rail system in high-speed Diesel engines is presented. The verification of the model was done in correlation to the results obtained on the real engine. Analysis represented preliminary results of mathematical simulation for Common Rail system gives good reason to ascertain that results correctly reproduced character of courses.*

### **1. Principle of common rail fuel system operation**

A specific of the Common Rail system is the use of a pipe (so-called rail), which is common to all individual fuel injectors and where the high-pressure fuel is kept by a control valve pump. The fuel injection is made by injectors with solenoid valves. In the Common Rail fuel injection system the fuel circulates in two circuits: a low-pressure one and a high-pressure one. The low-pressure circuit consists of a fuel tank, a fuel feed pump with a preliminary filter, low-pressure pipes and a fuel filter. The high-pressure circuit consists of a high-pressure pump a high-pressure line with a pressure sensor, a pressure regulator valve, a flow stopper and an injector [1,5,6].

At the moment of engine start the supply pump begins to force a fuel from the tank through the filter into the high-pressure pump. It works non-stop, independently of engine speed. Excess fuel flows off through the overflow-valve back to the tank.

High-pressure pump generates of constant pressure in high-pressure accumulator. The pump is driven by the clutch, gear or chain or cogbelt with half engine speed (max. to 3000 rpm). The fuel fed by the pump is forced across the filter with water settling into safety valve. Driving shaft by means of cams moves the pistons of pumping section. When the pressure will exceed opening pressure of safety-valve, feeding pump forces the fuel through the inflow valve of high pressure pump into chambers of pumping section, and the piston moves downwards (suction stroke). At the moment when piston reaches bottom dead centre inflow valve closes and the fuel cannot flow out. The fuel pressure generated by piston upwards movement opens outflow valve and compressed fuel goes to the high pressures circuit. The fuel is pumped so long, till the piston will reach top dead centre (delivery stroke). Then the pressure drops and outflow valve closes. The piston moves in suction stroke and residual fuel expands to the moment, when the pressure in pumping chamber has dropped below of pumping pressure of feeding pump, then inflow valve opens again and process goes on again.

Because high-pressure pump has a large delivery, at low and medium engine load excess fuel appears. Excess fuel is drained back to the tank through the pressure regulation valve. Its task is setting and keeping suitable pressure in the rail in dependences on engine load.

From high-pressure pump the fuel being under high pressure gets to the high-pressure



accumulator (the rail). All rail volume is constantly filled by fuel fed from the pump at pressure set by pressure regulating valve. Fuel compressibility reached as result of high pressure is used for obtainment of storing up effect. So, if from the rail the fuel is taken out for injection, the pressure in the rail remains nearly unchanged.

From the rail the fuel being under suitable pressure, in suitable quantity and at suitable moment gets to the injectors. On the way of fuel, between the rail and injector there is flow limiter, which performs essential part. The fuel received from the rail flows through hole and contraction in the limiter piston. In result the piston is moved by flowing fuel in direction of exit to the injector. If on both sides of the piston is the same pressure, piston movement is caused only by fuel flow and is limited by spring force. By reason of presence of fuel on injector side fuel pumping is not stopped. So, at normal work of injector the flow limiter is passive element. While the piston will move too far to injector line side, sealing piston face obturates limiter outlet and fuel pumping to the injector is broken. The fuel cannot flow and pressure on injector side drops. High differential pressure on inflow and outflow sides of the limiter causes the piston staying in tight position. So long as working pressure in the rail is present, the piston stays in this position. Not till then engine stopped the pressure in the rail drops and limiter spring force causes dislocation of the piston to its home position.

The injector controlled by the electromagnet consists of two parts:

- steering solenoid valve, consisted of coil and controlled valve,
- sprayer actuator, consisted of bar, needle and nozzle with 5 holes.

Fuel volume inside of controlled valve and immediately before actuator is called steering volume and it realizes basic part in injector working.

## **2. Regulation of injection dose and injection advance**

Fuel amount injected at every injection depends mostly on two parameters: opening time duration of the sprayer needle and injection pressure.

In the first approximation, it can be accepted, that pressure of injection dose is equal to pressure in whole system. But during injection not large pressure drop occurs. Because of difficult accessibility and short time it is no possibility of controlling and measuring pressure in the feeding chamber, that's why the pressure measured in fuel collector (rail) is taken as injection pressure.

Opening time of sprayer needle depends on time of electric power supply ET (Energizing Time). The longer time of electric steering responds to the longer time of armature opening and longer time of opening sprayer needle.

Effective injection time is not connected immediately with time of electric supply. Effective injection time is longer than time of electric steering, because delay time of injection end (TRFI) is longer than delay time of injection beginning (TRII). Delay between beginning of electric power supply and effective injection should be taken into consideration, if effective injection time be going to analyze. In reality, Common Rail steering system overtakes of injection electric steering in relation to effective beginning of injection. The same remarks refer to effective end of injection, which delay from the end of electric steering TRFI is longer than delay of injection beginning TRII. In order to know true fuel amount introduced during every injection, investigations for efficiency determination of injectors depending on electric steering time and different injection pressures should be done [2,4].

## **3. Basic assumptions for mathematical model**

Mathematical model of fuel injection system includes description of geometrical and physical features and mathematical dependences characterizing course of hydraulic

phenomena and dynamics of moving elements. Effectiveness of injection system mathematical model depends on goodness of mapping of real occurrences running in pump system, high-pressure pipeline and injector.

In order to most precise mapping of injection rates following assumptions were taken [3,4]:

- the fuel (gas oil) is a compressible liquid and is subject to Hooke's law,
- fuel flow in injection pipeline is treated as one-dimension flow; it results from that all parameters characterizing of the flow are directed along pipeline,
- fuel flow in injection system is treated as isothermal flow,
- influence of hydraulic resistance during fuel flow through the injection pipeline is taken into account, with assumption that losses during transient flow are the same as the losses for stabilized flow, for the same flow velocities and liquid proprieties; in equations describing fuel flow in pipeline the losses are formulated by coefficient of hydraulic resistances determined from dependence of Darcy-Weisbach,
- motion resistances of moving elements (bucket valve head, injector needle) are taken into account,
- damping and elasticity of bearing surfaces of bucket valve head and injector needle are taken into account,
- movements of moving elements of injection system are subject to action of inertial forces, damping and forces caused with liquid pressure and influences of springs in the system,
- physical parameters characterizing fuel, like modulus of elasticity, mass density, viscosity are the parameters depending on pressure and temperature,
- fuel leakages in the pumping system and injector are determined by viscosity coefficients depended on flow area and liquid proprieties,
- possibility of formation „empty spaces" in chamber of the bucket valve and injector is taken into account.

#### 4. Physical model of analyzed system and choice of computational method

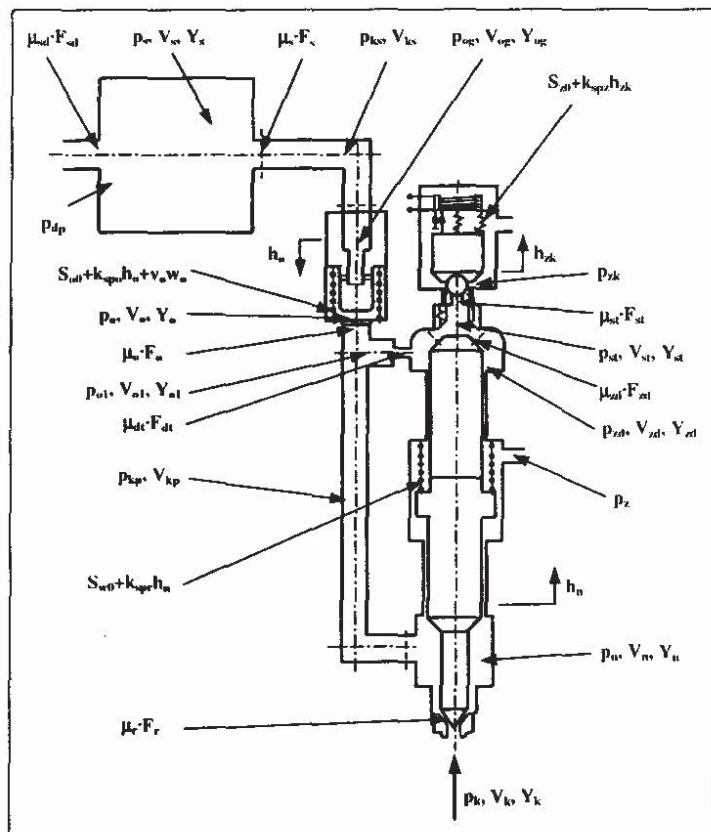


Fig. 1. Physical model of analyzed Common Rail system



Calculations refer to injection system used in engine of Fiat BRAVA 1,9 JTD 105 SX. Swept volume of the engine amounts to 1900 cm<sup>3</sup>, max power 77 [kW] at 4000 [rpm], instead max. torque 200 [N·m] at 1500 [rpm].

With assumption, that high pressures pump will pump the fuel at stable pressure, in the model following elements will be examined (fig. 1):

- fuel rail,
- injection pipelines,
- flow limiters,
- electronically controlled injectors.

The basic criterion at a choice of computational method is mapping exactitude of true courses of injection time characteristics. It depends on many factors, e.g. on manner of mathematical description of proceeded occurrences in injection system, on used methods of differential equations dissolving or also on accepted computational step.

Being governed by above-mentioned attentions, for realization hydrodynamic model was chosen. For description of fuel flow in pipeline the "moving volumetric element" method was used. It permits on reduction of equations number at simultaneous taking cavitation occurrences into consideration, what makes shorter time of calculations [3,4].

## 5. Differential equations relating to Common Rail system

Mathematical equations describing course of occurrences in the system have a special meaning in computational method, because they determine time- and local course of state parameters at fuel flow through the injection system. Exactitude of all computational methods depend on precision of their formulation.

Differential equations describing hydrodynamic occurrences for the Common Rail system have following form:

- the rail

$$\frac{dY_{sr}}{d\varphi} = \left( \mu_{sd} F_{sd} \sqrt{\frac{2}{\rho_{pul}} |p_{dp} - p_s|} - C_{sl} \cdot F_s \right) \cdot \frac{1}{\omega} \quad (1)$$

- high pressures pipeline behind the rail

$$\left. \begin{aligned} \frac{dC_j}{d\varphi} &= \left( \frac{F_s (p_{j-1} - p_j)}{m_j} - 2 \cdot K_p \cdot C_j \right) \cdot \frac{1}{\omega} \\ \frac{dY_j}{d\varphi} &= F_s \cdot (C_{sj} - C_{sj+1}) \cdot \frac{1}{\omega} \end{aligned} \right\} \text{for } j = 1, 2, \dots, k_s \quad (2)$$

- flow limiter

$$\frac{dC_{og}}{d\varphi} = \left( \frac{F_s \cdot (p_{ks} - p_{og})}{m_{ks}} - 2 \cdot K_p \cdot C_{og} \right) \cdot \frac{1}{\omega} \quad (3)$$

$$\frac{dY_{og}}{d\varphi} = \left( F_s \cdot C_{og} - \text{sgn}(p_{og} - p_o) \cdot \mu_{od} \cdot F_{od} \cdot \sqrt{\frac{2}{\rho_{pul}} |p_{og} - p_o|} - v_o \cdot A_o \right) \cdot \frac{1}{\omega} \quad (4)$$

$$\frac{dY_o}{d\varphi} = \left( \text{sgn}(p_{og} - p_o) \cdot \mu_{od} \cdot F_{od} \cdot \sqrt{\frac{2}{\rho_{pul}} |p_{og} - p_o|} + v_o \cdot F_o - \mu_o \cdot C_o \cdot F_o(h_o) \right) \cdot \frac{1}{\omega} \quad (5)$$

$$C_o = \text{sgn}(p_o - p_{o1}) \cdot \sqrt{\frac{2}{\rho_{put}} |p_o - p_{o1}|} \quad (6)$$

$$\frac{dh_o}{d\varphi} = \frac{v_o}{\omega} \quad (7)$$

$$\begin{aligned} \frac{dV_o}{d\varphi} = & [A_o(h_o) \cdot p_{oz} - A_o(h_o) \cdot p_o - (S_{o0} + h_o \cdot k_{spo} + v_o \cdot w_o) - B_{o1} \cdot (w_{zo} \cdot v_o + k_{zo} \cdot h_o) + \\ & - B_{o2} \cdot (w_{zo} \cdot v_o + k_{zo} \cdot h_o)] \cdot \frac{1}{m_o \omega} \end{aligned} \quad (8)$$

$$\frac{dY_{o1}}{d\varphi} = (\mu_o \cdot C_o \cdot F_o(h_o) - \mu_{d1} \cdot F_{d1} - C_{p1} \cdot F_p) \cdot \frac{1}{\omega} \quad (9)$$

- feeding line before the injector

$$\left. \begin{aligned} \frac{dC_i}{d\varphi} &= \left( \frac{F_p(p_{i-1} - p_i)}{m_i} - 2 \cdot K_p \cdot C_i \right) \cdot \frac{1}{\omega} \\ \frac{dY_i}{d\varphi} &= F_p \cdot (C_i - C_{i+1}) \cdot \frac{1}{\omega} \end{aligned} \right\} \text{for } i = 1, 2, \dots, k_p \quad (10)$$

$$\frac{dC_n}{d\varphi} = \left( \frac{F_p(p_{kp} - p_n)}{m_{kp}} - 2 \cdot K_p \cdot C_n \right) \cdot \frac{1}{\omega} \quad (11)$$

- sprayer chamber

$$\frac{dY_n}{d\varphi} = (F_p \cdot C_n - Q_n - F_b \cdot v_n - Q_p) \cdot \frac{1}{\omega} \quad (12)$$

$$\frac{dQ}{d\varphi} = \frac{Q_{n1}}{\omega} \quad (13)$$

$$Q_{n1} = B_{n3} \cdot \mu_r \cdot F_r(h_n) \cdot \sqrt{\frac{2}{\rho_{put}} |p_n - p_k|} \quad (14)$$

$$\begin{aligned} \frac{dV_n}{d\varphi} = & [A_b \cdot p_n + A_k \cdot p_k - (A_{vst} \cdot p_{st} + A_{zd} \cdot p_{zd}) - S_{n0} + k_{spr} \cdot h_n - w_n \cdot v_n + \\ & - B_{n1} \cdot (w_{gn} \cdot v_n + k_{gn} \cdot h_n) - B_{n2} \cdot (w_{zn} \cdot v_n + k_{zn} \cdot (h_n - h_{max}))] \cdot \frac{1}{m_n \cdot \omega} \end{aligned} \quad (15)$$

$$\frac{dh_n}{d\varphi} = \frac{v_n}{\omega} \quad (16)$$

- chamber over the needle bar

$$\frac{dY_{zd}}{d\varphi} = (\mu_{d1} \cdot F_{d1} \cdot C_{d1} + A_z \cdot h_n - \mu_{zd} \cdot F_{zd}(h_n) \cdot C_{zd}) \cdot \frac{1}{\omega} \quad (17)$$

$$C_{d1} = \text{sgn}(p_{o1} - p_{zd}) \cdot \sqrt{\frac{2}{\rho_{put}} |p_{o1} - p_{zd}|} \quad (18)$$

$$C_{zd} = \text{sgn}(p_{zd} - p_{st}) \cdot \sqrt{\frac{2}{\rho_{pal}} |p_{zd} - p_{st}|} \quad (19)$$

- chamber below of ball valve

$$\frac{dY_{st}}{d\varphi} = (\mu_{zd} \cdot F_{zd}(h_n) \cdot C_{zd} - \mu_{st} \cdot F_{st} \cdot C_{st} - A_{zk} \cdot v_{zk}) \cdot \frac{1}{\omega} \quad (20)$$

$$C_{st} = \text{sgn}(p_{st} - p_{zk}) \cdot \sqrt{\frac{2}{\rho_{pal}} |p_{st} - p_{zk}|} \quad (21)$$

$$\frac{dh_{zk}}{d\varphi} = \frac{v_{zk}}{\omega} \quad (22)$$

$$\begin{aligned} \frac{dv_{zk}}{d\varphi} = & \left[ A_{ku} \cdot p_{st} - A_{zk} \cdot p_{zk} - S_{z0} + k_{spz} \cdot h_z - w_{zk} \cdot v_k + F_c + \right. \\ & \left. - B_{zk1} \cdot (w_{gzk} \cdot v_{zk} + k_{gzk} \cdot h_{zk}) - B_{n2} (w_{zrk} \cdot v_{zk} + k_{zrk} \cdot (h_{zk} - h_{zk\max})) \right] \cdot \frac{1}{m_{zk} \cdot \omega} \end{aligned} \quad (23)$$

- where:

$$F_n(h_n) = \pi \cdot h_n \cdot \sin \frac{\alpha_n}{2} \cdot \left( d_p - h_n \cdot \sin \frac{\alpha_n}{2} \cdot \cos \frac{\alpha_n}{2} \right) [\text{mm}^2] \quad (24)$$

$$F_p = \pi \cdot \left( \frac{d_p}{2} \right)^2 [\text{mm}^2] \quad (25)$$

$$F_r(h_n) = \pi \cdot h_n \cdot \sin \frac{\alpha_r}{2} \cdot \left( d_s - h_n \cdot \sin \frac{\alpha_r}{2} \cdot \cos \frac{\alpha_r}{2} \right) [\text{mm}^2] \quad (26)$$

$$F_s = \pi \cdot \left( \frac{d_s}{2} \right)^2 [\text{mm}^2] \quad (27)$$

$$F_{zd}(h_n) = \pi \cdot h_n \cdot \sin \frac{\alpha_z}{2} \cdot \left( d_{st} - h_n \cdot \sin \frac{\alpha_z}{2} \cdot \cos \frac{\alpha_z}{2} \right) [\text{mm}^2] \quad (28)$$

$$F_{st} = \pi \cdot \left( \frac{d_{st}}{2} \right)^2 [\text{mm}^2] \quad (29)$$

- |                                |  |
|--------------------------------|--|
| $h_0, h_n, h_{zd}$             | - jump of the the flow limiter piston, the sprayer needle, the needle bar,             |
| $d_p, d_s, d_{st}$             | - diameter of the high pressures pipeline, the sac hole, the steering channel,         |
| $\alpha_n, \alpha_r, \alpha_z$ | - angle of the limiter cone, the needle, the bar,                                      |
| $A$                            | - refers to all surfaces,  |
| $F$                            | - refers to flow areas,  |
| $\rho_{pal}$                   | - fuel density,  |
| $p_s, p_{og}, p_0$             | - pressure in the rail, before the flow limiter, beyond the flow limiter,              |
| $p_j, p_i$                     | - pressure in a given section of the pipeline behind the rail and before the injector, |
| $C_j, C_i$                     | - fuel velocity in a given area (as above),  |
| $\varphi, \omega$              | - angle of rotation and angular velocity,  |
| $m_i, m_j$                     | - fuel mass in a suitable pipeline fragment,   |
| $\mu$                          | - factor of flow rate,   |
| $F_s$                          | - area of the pipeline behind the rail,  |
| $F_p$                          | - flow area of feed line of the injector,  |

$v_o, h_o$	- speed and jump of the discharge valve,
$v_{zk}, h_{zk}$	- speed and jump of the ball valve,
$v_n, h_n$	- speed and jump of the sprayer needle,
$K_p$	- coefficient of fuel flow resistance in the pipeline,
$k_{gn}, k_{zn}$	- elasticity of the sprayer seat and bumper,
$\mu \cdot F_o(h_o)$	- effective area of fuel outflow from the chamber of flow limiter,
$\mu \cdot F_{zd}(h_n)$	- effective area of fuel outflow from the steering chamber,
$\mu \cdot F_r(h_r)$	- effective area of the sprayer in function of needle jump,
$Y_{so}, Y_{og}, Y_o, Y_n$	- volumetric variables in the chambers of rail, the upper chamber of flow limiter, the bottom chamber of flow limiter, the sprayer,
$Y_{zd}, Y_{st}, Y_j, Y_i$	- volumetric variables in the chambers over needle bar, under ball valve, in j-teenth section of the pipeline behind the rail and i-teenth section of the pipeline before the injector,
$Q_n$	- stream of fuel flowing out from sprayer hole,
$B_n$	- steering coefficients for the sprayer: $B_{n1} = 0$ for $h_n > 0$ ; $B_{n1} = 1$ for $h_n \leq 0$ $B_{n2} = 0$ for $h_n < h_{nmax}$ ; $B_{n2} = 1$ for $h_n \geq h_{nmax}$ $B_{n3} = 1$ for $P_n > P_k$ ; $B_{n3} = 0$ for $P_n \leq P_k$
$B_o$	- steering coefficients for the flow limiter: $B_{o1} = 0$ for $h_o > 0$ ; $B_{o1} = 1$ for $h_o \leq 0$ $B_{o2} = 0$ for $h_o < h_{omax}$ ; $B_{o2} = 1$ for $h_o \geq h_{omax}$

## 6. Results of investigations and recapitulation

Presented mathematical model and on its base worked out simulation program permit on determination of characteristics of injection rate in the Common Rail systems. Indispensable constructional data were obtained by measuring of original parts of the system used in engine of Fiat BRAVA 1,9 JTD 105 SX vehicle.

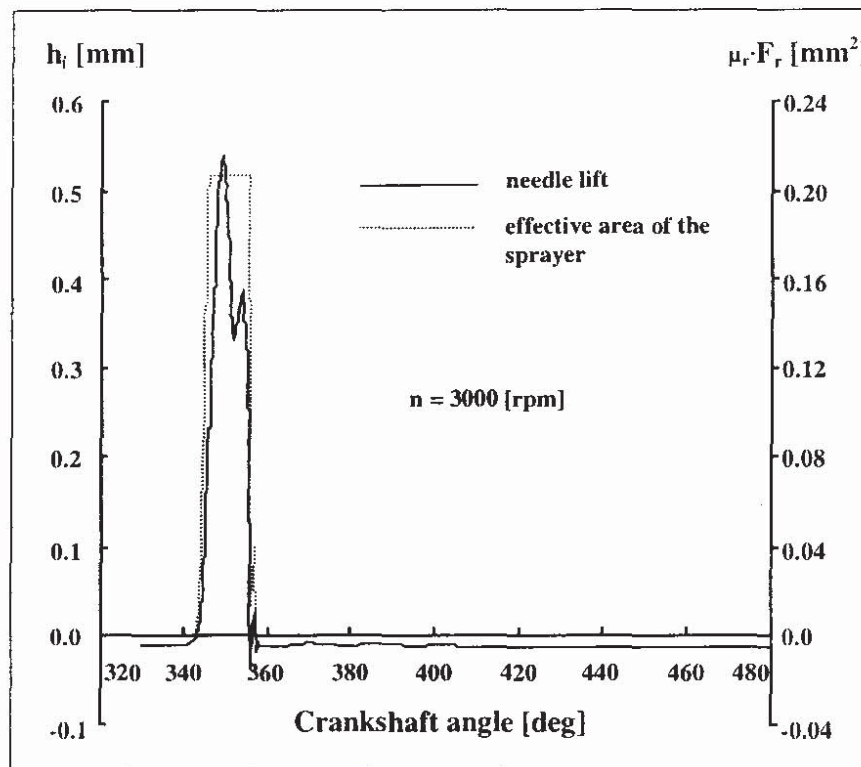


Fig. 2. Rate of needle lift ( $h_i$ ) and change of sprayer effective area ( $\mu_r \cdot Fr$ )



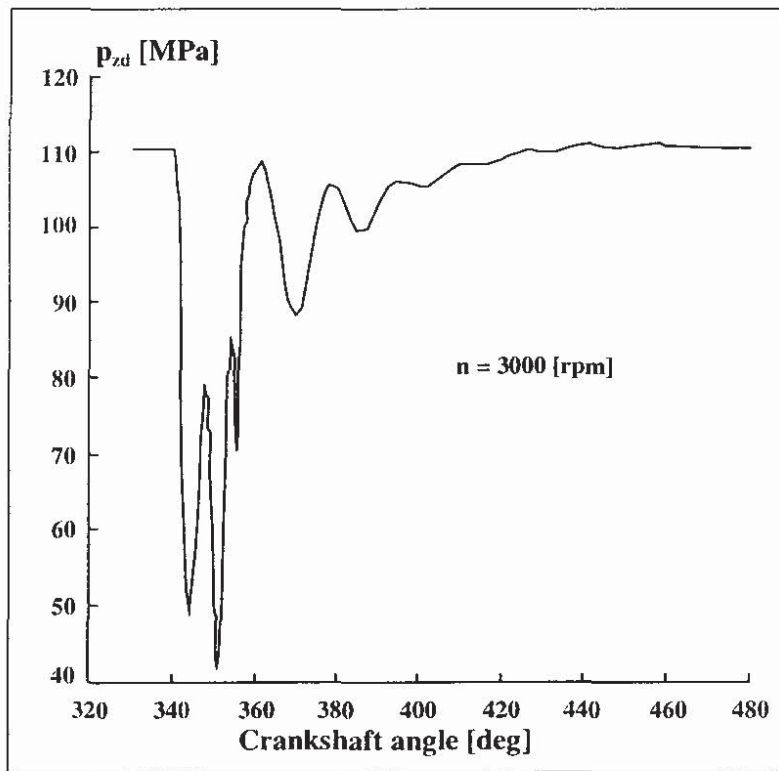


Fig. 3. Rate of pressure in chamber over the needle ( $p_{zd}$ )

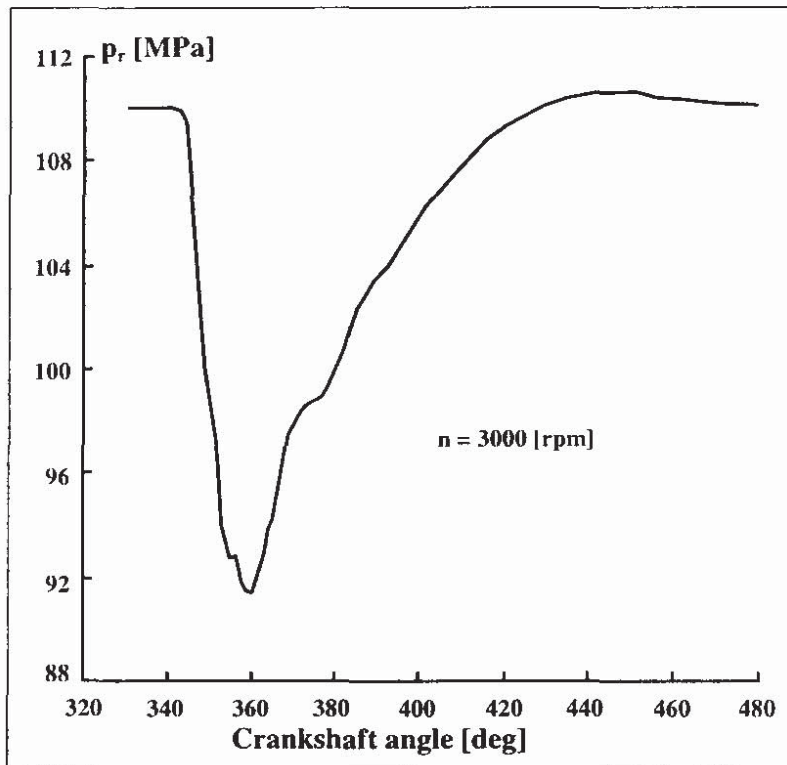


Fig. 4. Rate of pressure in sprayer chamber ( $p_r$ )



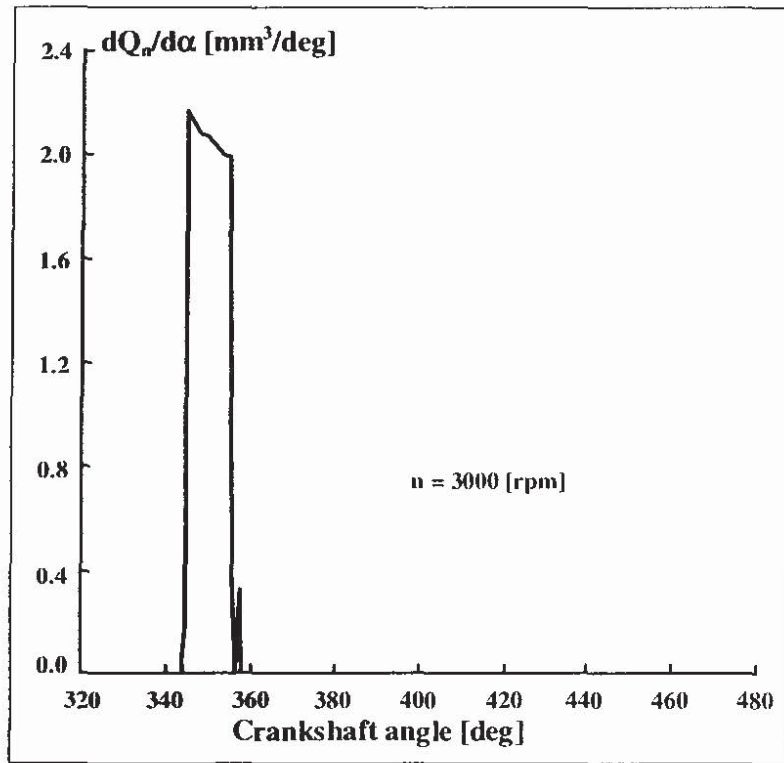


Fig. 5. Change of fuel dose flowed out from the sprayer ( $Q_n$ )

In a result of computational experiment characteristics describing courses of basic steering, feeding and of fuel injection parameters were received. Besides approximate information about quantitative and qualitative influence of physical fuel properties, constructional features of system elements, time of steering current impulse and initial values of fuel pressure on injection process were obtained.

Example results of computer simulation are presented for rotational speeds of engine crankshaft amount to  $n = 3000$  [rpm] and it is shown adequately on figs. 2, 3, 4, 5.

Verification of the results was done for injector needle lift and injected fuel doses. The curves show, that satisfactory agreement both for character of courses and numerical values was received.

## References

- [1] Dutko R. , Lejda K.: Design and operation of Common Rail system in high-speed Diesel Engines. Western Scientific Centre of Ukrainian Transport Academy, No 9, Lvov 2002.
- [2] Janiszewski T., Spiros M.: Elektroniczne układy wtryskowe silników wysokoprężnych. WKŁ, Warszawa 2001.
- [3] Lejda K.: Numerical simulation of fuel injection process in the aspect of selection of injection feed system elements for Diesel engine. Proceedings Intern. AMSE Conference on "Signals, Data, Systems", Vol. 2, Chicago (USA) 1992.
- [4] Lejda K.: Selected problems of fuel supply in high-speed Diesel engines. Publishers "META", Lvov 2004.
- [5] Technical guide of Bosch: Układy wtryskowe Common Rail. WKŁ, Warszawa 2000.
- [6] Materiały informacyjne firm Bosch, Renault, Mercedes, Citroën, Deteq.